The Fundamental Interactions after the Higgs Discovery

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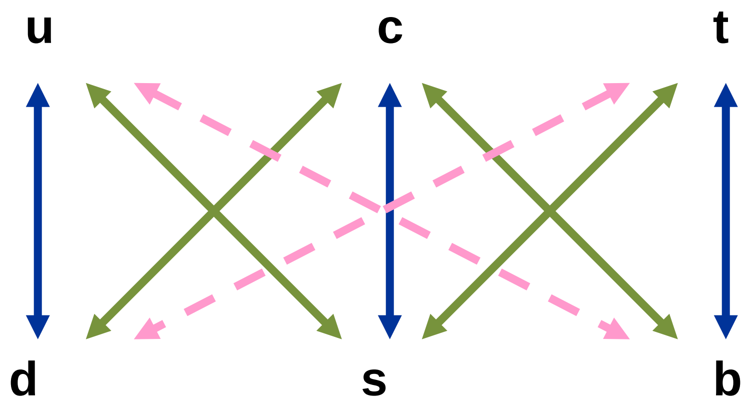
arXiv:1201.0537 (Standard Model)

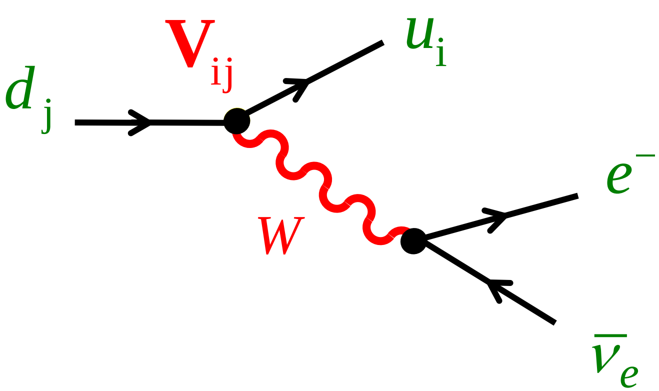
arXiv:1512.08749 (Higgs Physics)

arXiv:1805.08597 (Flavour Physics)

Escuela Avanzada de Verano, México, 19-23 July 2021

Flavour Changing Charged Currents





$$\Gamma(d_j \to u_i\, e^-\, \overline{\nu}_e) \propto \left|\mathbf{V}_{ij}\right|^2$$

We measure decays of hadrons (no free quarks)

Important QCD Uncertainties

V_{ij} Determinations

PDG 2020

CKM entry	Value	Source
$\lvert V_{ud}\rvert$	0.97370 ± 0.00014	Nuclear β decay
$\lvert V_{us}\rvert$	0.2245 ± 0.0008	$K \to (\pi)\,\ell\,\nu$
$\lvert V_{cd}\rvert$	0.221 ± 0.004	$D \to (\pi)\,\ell\,\nu\,,\ \nu\, d \to c\, X$
$\lvert V_{cs}\rvert$	0.987 ± 0.011	$D \to K\,\ell\,\nu\,,\ D_s \to \ell\,\nu$
$\lvert V_{cb}\rvert$	0.0410 ± 0.0014	$b \to c\,\ell\,\nu\,,\ B \to D^{(*)}\ell\,\nu$
$\lvert V_{ub}\rvert$	0.00382 ± 0.00024	$b \to u\,\ell\,\nu\,,\ B \to \pi\,\ell\,\nu$
$\lvert V_{tb}\rvert$	1.013 ± 0.030	$p\bar{p},\ p\,p \to tb + X$
$\lvert V_{tb}\rvert \Big/ \sqrt{\sum_q \lvert V_{tq}\rvert^2}$	$>\ 0.975$ (95% CL)	$t \to bW\, /\, t \to qW$
$\lvert V_{td}\rvert$	0.0080 ± 0.0003	$B_d^0 - \bar{B}_d^0$ mixing
$\lvert V_{ts}\rvert$	0.0388 ± 0.0011	$B_s^0 - \bar{B}_s^0$ mixing

PDG 2018

0.97420 (21)

0.2243 (5)

0.9994 ± 0.0005 (PDG 2018)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985 \pm 0.0005$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.025 \pm 0.022$$

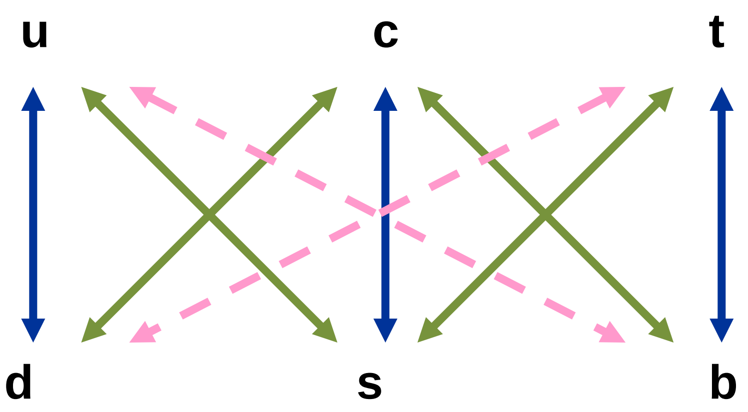
$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1.028 \pm 0.061$$

$$\sum_j \left(|V_{uj}|^2 + |V_{cj}|^2 \right) = 2.002 \pm 0.027 \quad \text{(LEP)}$$

Hierarchical Structure

$$\mathbf{V} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$$\lambda \approx \sin\theta_C \approx 0.225 \quad ; \quad A \approx 0.81 \quad ; \quad \sqrt{\rho^2+\eta^2} \approx 0.4$$



QUARK MIXING MATRIX

- **Unitary** $N_G \times N_G$ **Matrix:** N_G^2 **parameters**

$$\mathbf{V} \cdot \mathbf{V}^\dagger = \mathbf{V}^\dagger \cdot \mathbf{V} = \mathbf{1}$$

$\frac{1}{2} N_G (N_G - 1)$ moduli, $\frac{1}{2} N_G (N_G + 1)$ phases

- $2 N_G - 1$ **arbitrary phases:** $\bar{u}_i \mathbf{V}_{ij} d_j$

$$u_i \to e^{i\phi_i} u_i \quad ; \quad d_j \to e^{i\theta_j} d_j \quad \Longrightarrow \quad \mathbf{V}_{ij} \to e^{i(\theta_j - \phi_i)} \mathbf{V}_{ij}$$

$\Downarrow$

$\mathbf{V}_{ij}$ **Physical Parameters:**

$\frac{1}{2} N_G (N_G - 1)$ **moduli** ; $\frac{1}{2} (N_G - 1)(N_G - 2)$ **phases**

- **$N_f = 2$: 1 angle, 0 phases** **(Cabibbo)**

$$\mathbf{V} = \begin{bmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{bmatrix} \quad \Longrightarrow \quad \text{No } \cancel{\mathcal{CP}}$$

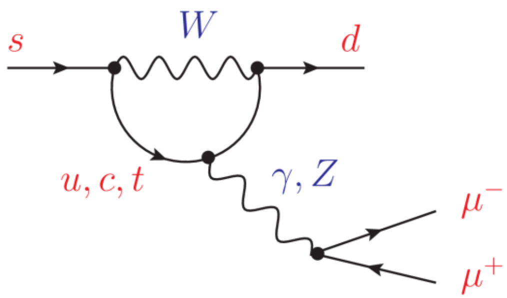
- **$N_f = 3$:** 3 angles, 1 phase **(CKM)** $\quad c_{ij} \equiv \cos\theta_{ij} \quad ; \quad s_{ij} \equiv \sin\theta_{ij}$

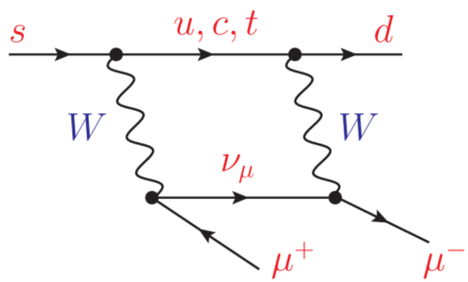
$$\mathbf{V} = \begin{bmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta_{13}} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta_{13}} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta_{13}} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta_{13}} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta_{13}} & c_{23}\,c_{13} \end{bmatrix}$$

$$\approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$\lambda \approx \sin\theta_C \approx 0.225 \quad ; \quad A \approx 0.81 \quad ; \quad \sqrt{\rho^2 + \eta^2} \approx 0.4$ $\qquad \delta_{13} \neq 0 \quad (\eta \neq 0) \quad \Longrightarrow \quad \cancel{\mathcal{CP}}$

GIM Mechanism





$$\mathcal{M} \propto \sum_{i=u,c,t} V_{is}\, V_{id}^*\, F(m_i^2/M_W^2)$$

$$\sum_{i=u,c,t} V_{is}\, V_{id}^* = 0 \quad \Rightarrow \quad \mathcal{M} = 0 \quad \text{if} \quad m_u = m_c = m_t$$

$$\tilde{F}(x) \equiv F(x) - F(0) \quad \Rightarrow$$

$$\boxed{\begin{aligned} \mathcal{M} &\propto V_{cs}\, V_{cd}^*\, \tilde{F}(m_c^2/M_W^2) + V_{ts}\, V_{td}^*\, \tilde{F}(m_t^2/M_W^2) \\ &\approx -\lambda\, \tilde{F}(m_c^2/M_W^2) - \lambda^5 A^2 (1-\rho+i\eta)\, \tilde{F}(m_t^2/M_W^2) \end{aligned}}$$

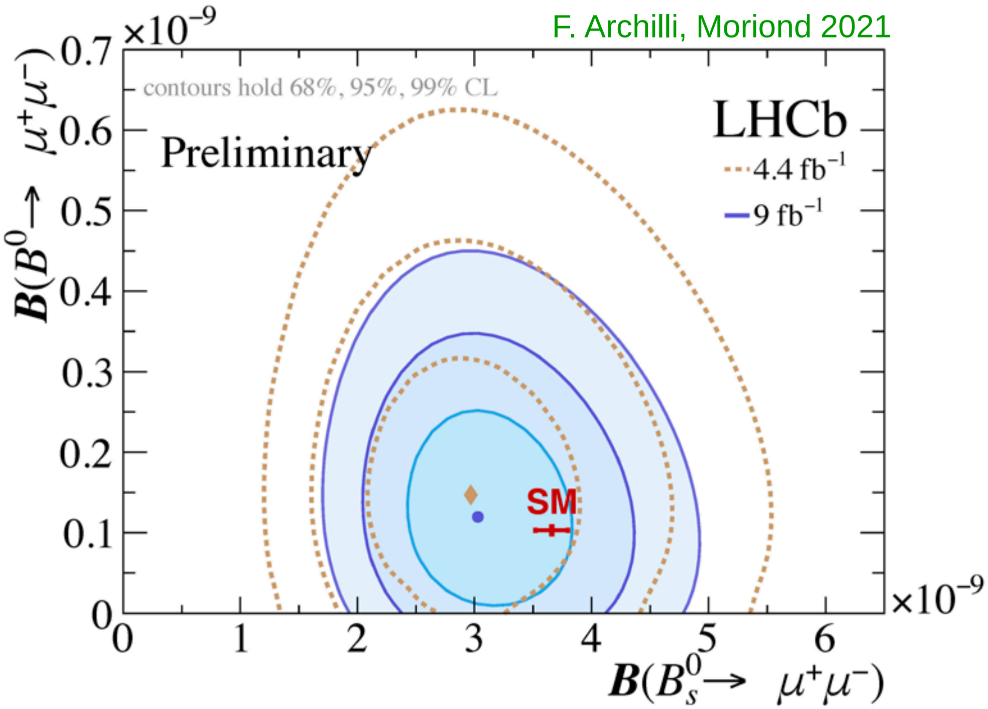
- **Top contribution dominates. Strong suppression:** $\mathcal{M} \propto \frac{g^4}{16\pi^2}\left[\lambda^5 A^2 \frac{m_t^2}{M_W^2}, \lambda \frac{m_c^2}{M_W^2}\right]$
- **$\not{C}$P effects fully governed by top contribution** $\left[\mathrm{Im}(V_{cs}\, V_{cd}^*) = -\,\mathrm{Im}(V_{ts}\, V_{td}^*)\right]$

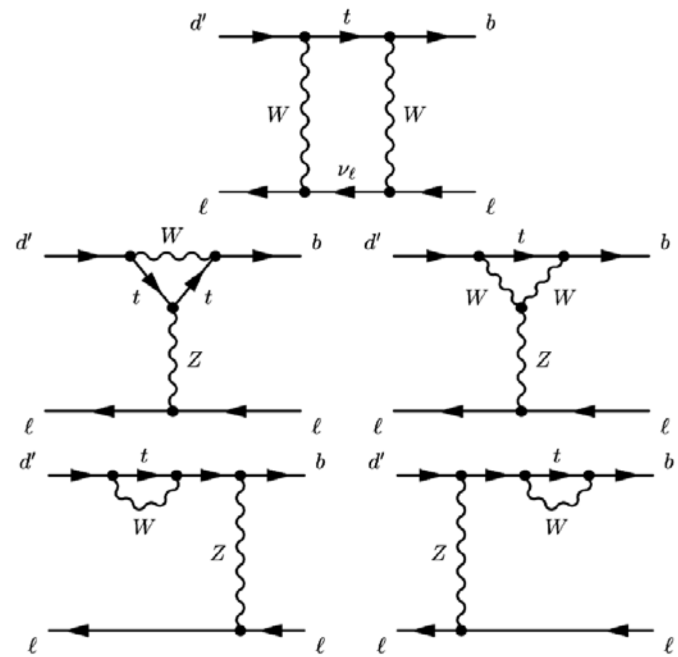
Rare Decays

Loop & CKM suppression
Sensitivity to New Physics

$B_{s,d} \to \mu^+\mu^-$

F. Archilli, Moriond 2021





$W^\pm \leftrightarrow H^\pm$, $Z \leftrightarrow H^0, A^0$

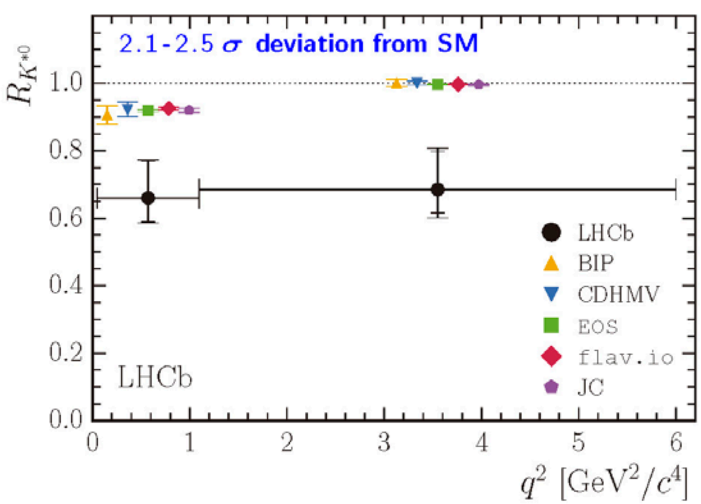
$$\bar{B}(B_s^0 \to \mu^+\mu^-)_{\text{exp}} = (2.9 \pm 0.4) \cdot 10^{-9}$$

$$\bar{B}(B_d^0 \to \mu^+\mu^-)_{\text{exp}} = \left(0.5\,^{+1.7}_{-1.5}\right) \cdot 10^{-10}$$

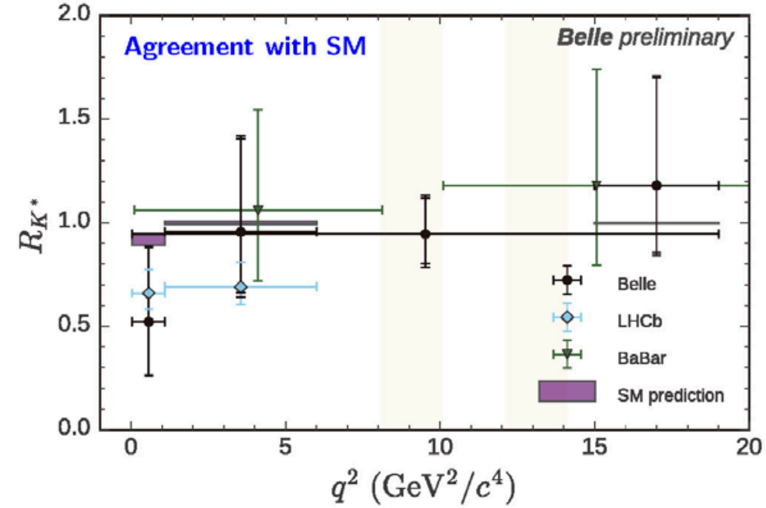
Sensitive to (pseudo) scalar contributions

Violations of Lepton Flavour Universality

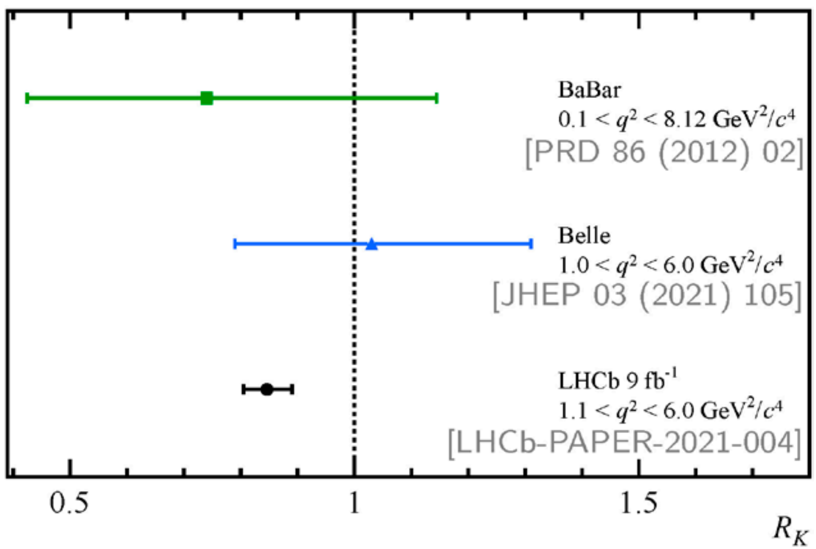
LHCb 1705.05802



Belle 1904.02440



$$\frac{\Gamma(B \to K^* \mu^+ \mu^-)}{\Gamma(B \to K^* e^+ e^-)}$$



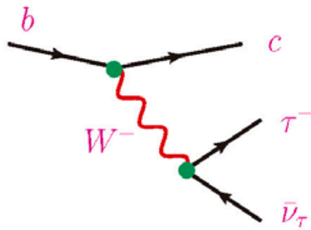
$$R_K \equiv \frac{\Gamma(B^+ \to K^+ \mu^+ \mu^-)}{\Gamma(B^+ \to K^+ e^+ e^-)} = 0.846 \,{}^{+0.042}_{-0.039}\,{}^{+0.013}_{-0.012}$$

3.1 σ below the SM

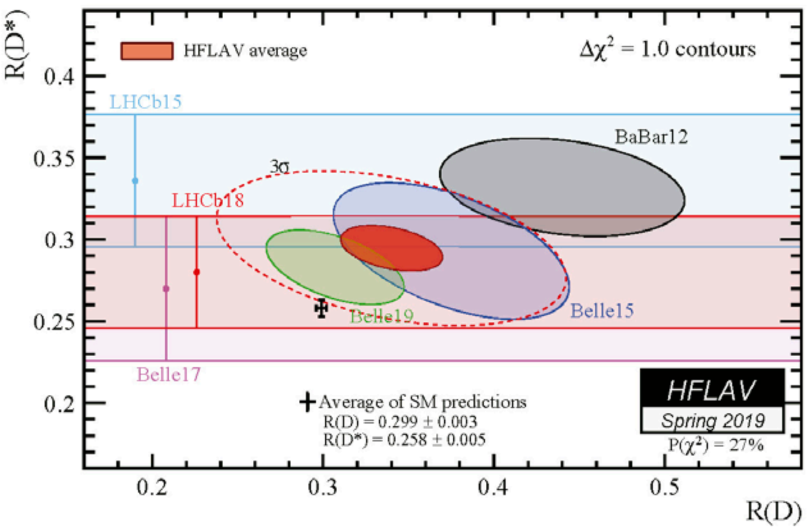
Flavour Anomaly (charged currents)

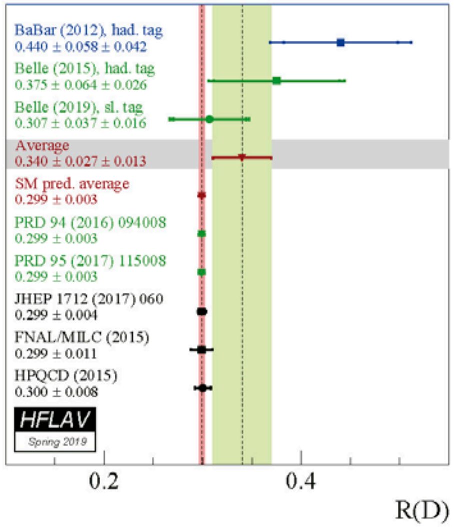
$$\mathcal{R}_{D^{(*)}} \equiv \frac{\mathcal{B}(\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D^{(*)}\ell^-\bar{\nu}_\ell)}$$

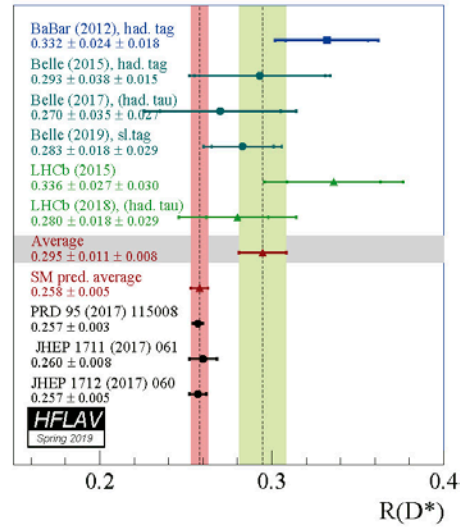
Tree-level process



3.08 σ discrepancy (3.2 σ with more recent predictions)



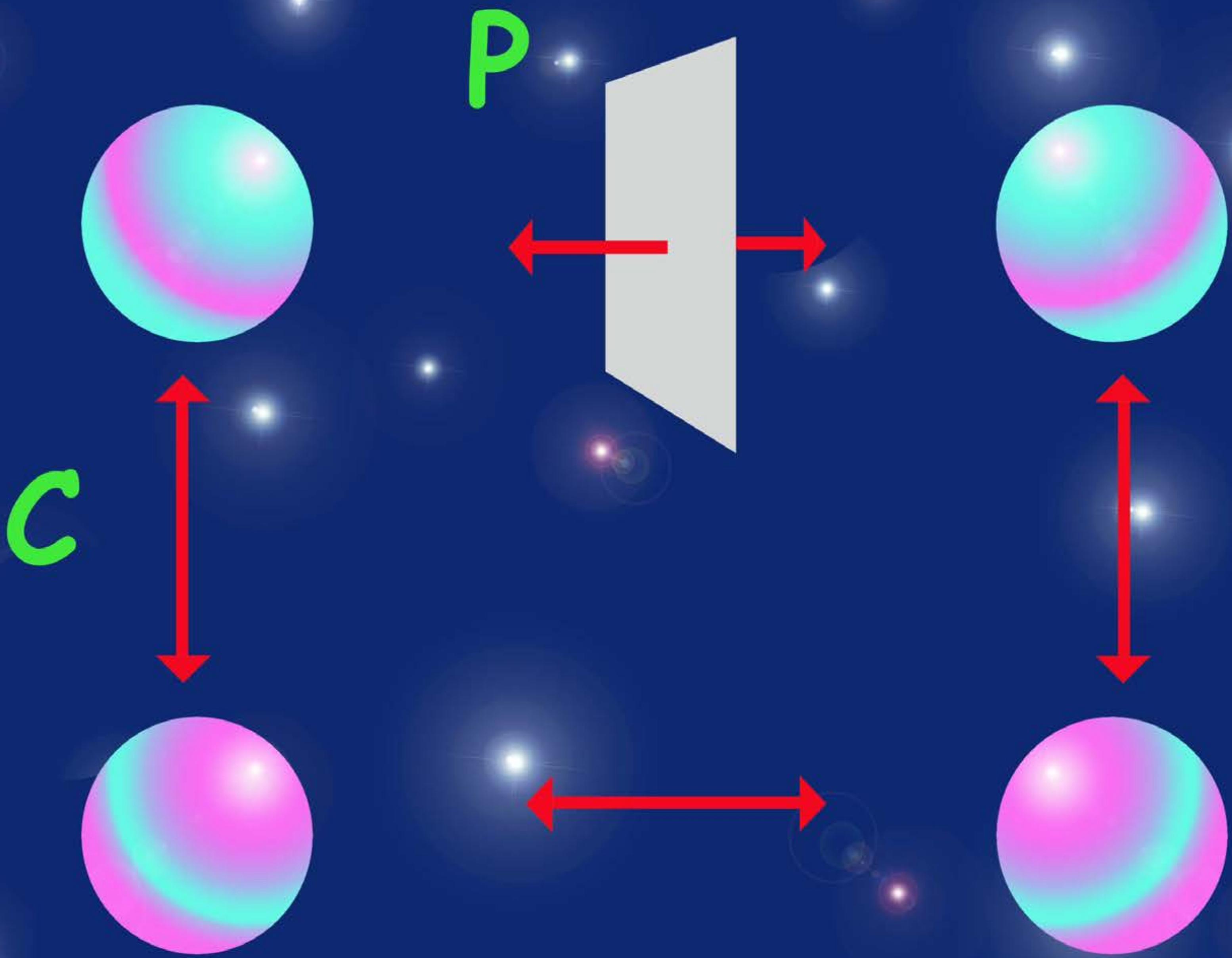




LHCb, 1711.05623: $\mathcal{R}_{J/\psi} \equiv \dfrac{\mathcal{B}(B_c \to J/\psi\tau\bar{\nu}_\tau)}{\mathcal{B}(B_c \to J/\psi\mu\bar{\nu}_\mu)} = 0.71 \pm 0.17 \pm 0.18$ (1.7 σ) $\mathcal{R}_{J/\psi}^{\mathrm{SM}} \approx 0.26 - 0.28$

Belle, 1903.03102: $F_L^{D^*} = 0.60 \pm 0.08 \pm 0.04$ (1.6 σ) $F_{L,\mathrm{SM}}^{D^*} = 0.455 \pm 0.003$

Belle, 1612.00529: $\mathcal{P}_\tau^{D^*} = -0.38 \pm 0.51^{+0.21}_{-0.16}$ $\mathcal{P}_{\tau,\mathrm{SM}}^{D^*} = -0.499 \pm 0.003$


P
C

- $\mathcal{C}$, $\mathcal{P}$: Violated maximally in weak interactions
- $\mathcal{CP}$: Symmetry of nearly all observed phenomena
- Slight (~ 0.2 %) $\cancel{\mathcal{CP}}$ in K^0 decays (1964)
- Sizeable $\cancel{\mathcal{CP}}$ in B^0 decays (2001)
- Huge Matter — Antimatter Asymmetry in our Universe

  $\Longrightarrow$ Baryogenesis

- Small $\cancel{\mathcal{CP}}$ in D^0 decays (LHCb, 2019)

$$\boxed{\mathcal{CPT}\ \textbf{Theorem:}\quad \cancel{\mathcal{CP}} \Longleftrightarrow \cancel{\mathcal{T}}}$$

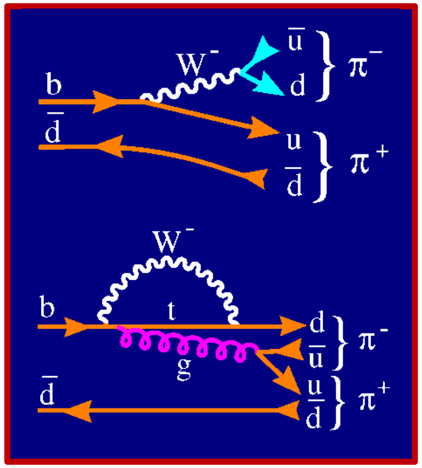
Thus, $\cancel{\mathcal{CP}}$ requires:

- Complex Phases
- Interferences

DIRECT $\not{\mathcal{CP}}$

$$\left|\mathbf{T}(\mathrm{P}\to\mathrm{f})\right| \neq \left|\mathbf{T}(\overline{\mathrm{P}}\to\overline{\mathrm{f}})\right|$$

$\overline{B}_d^0$



$$\mathbf{T}(\mathrm{P}\to\mathrm{f}) = \mathrm{T}_1\, e^{i\phi_1}\, e^{i\delta_1} + \mathrm{T}_2\, e^{i\phi_2}\, e^{i\delta_2}$$

$\downarrow \mathcal{CP}$

$$\mathbf{T}(\overline{\mathrm{P}}\to\overline{\mathrm{f}}) = \mathrm{T}_1\, e^{-i\phi_1}\, e^{i\delta_1} + \mathrm{T}_2\, e^{-i\phi_2}\, e^{i\delta_2}$$

$$\boxed{A^{\mathrm{CP}}_{\mathrm{P}\to\mathrm{f}} \equiv \frac{\Gamma(\mathrm{P}\to\mathrm{f})-\Gamma(\overline{\mathrm{P}}\to\overline{\mathrm{f}})}{\Gamma(\mathrm{P}\to\mathrm{f})+\Gamma(\overline{\mathrm{P}}\to\overline{\mathrm{f}})} = \frac{-2\,\mathrm{T}_1\,\mathrm{T}_2\,\sin(\phi_2-\phi_1)\,\sin(\delta_2-\delta_1)}{\mathrm{T}_1^{\,2}+\mathrm{T}_2^{\,2}+2\,\mathrm{T}_1\,\mathrm{T}_2\,\cos(\phi_2-\phi_1)\,\cos(\delta_2-\delta_1)}}$$

One needs:

- **2 Interfering Amplitudes**
- **2 Different Weak Phases** $\left[\sin(\phi_2-\phi_1)\neq 0\right]$
- **2 Different FSI Phases** $\left[\sin(\delta_2-\delta_1)\neq 0\right]$

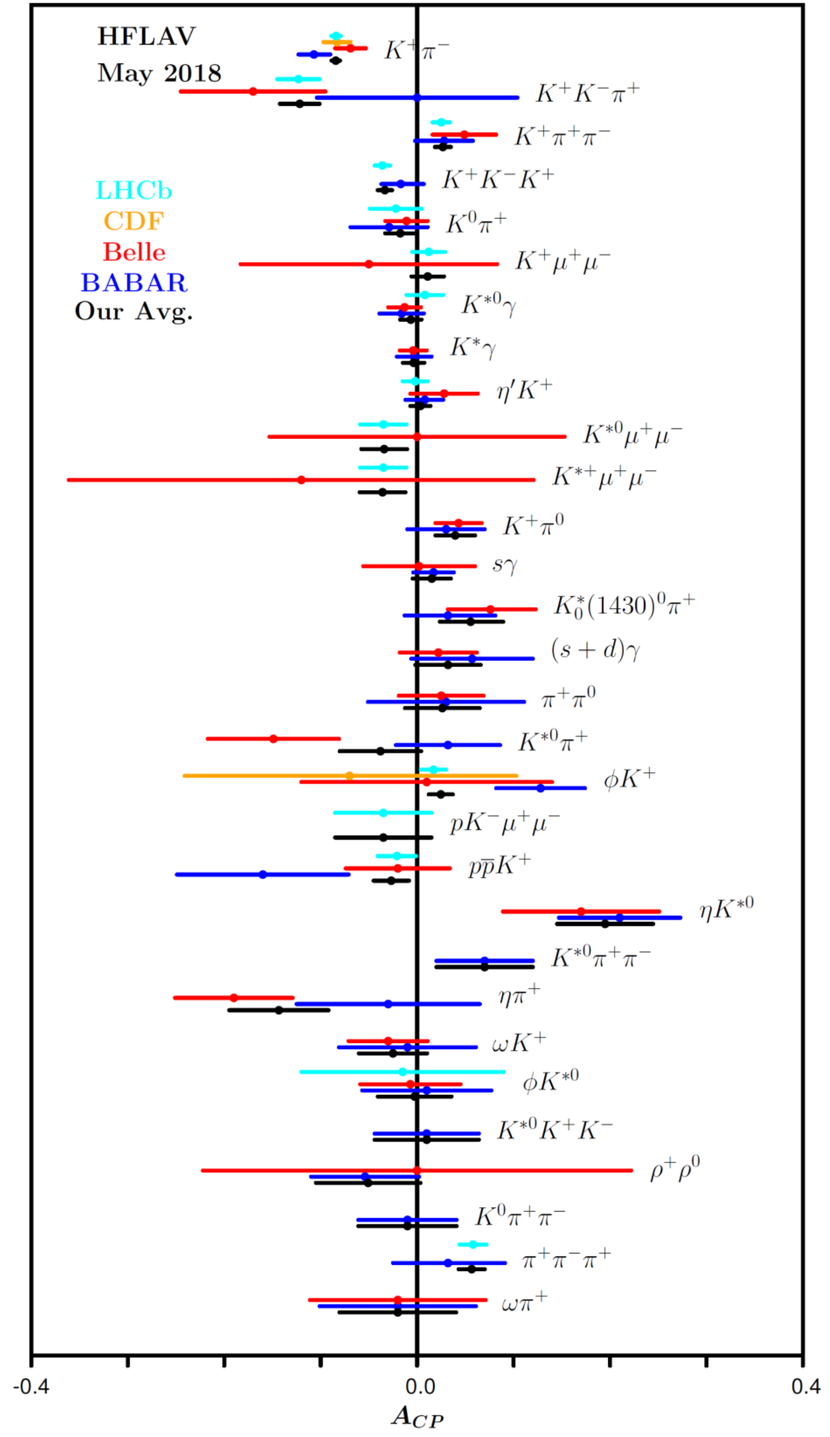
DIRECT $\cancel{CP}$

$$A_{CP}(B\to f)\equiv\frac{\mathrm{Br}(\bar{B}\to\bar{f})-\mathrm{Br}(B\to f)}{\mathrm{Br}(\bar{B}\to\bar{f})+\mathrm{Br}(B\to f)}$$

$$A_{CP}(B_d^0\to\pi^-K^+) = -0.084\pm0.004 \qquad \textbf{(21 σ)}$$

$$A(B_s^0\to\pi^-K^+) = -0.213\pm0.017 \qquad \textbf{(12.5 σ)}$$

$$A_{CP}(B^+\to K^+K^-\pi^+) = -0.122\pm0.021 \qquad \textbf{(5.8 σ)}$$

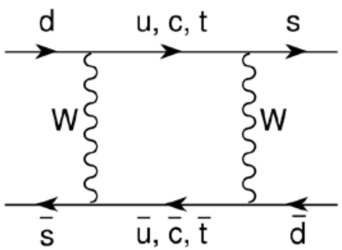


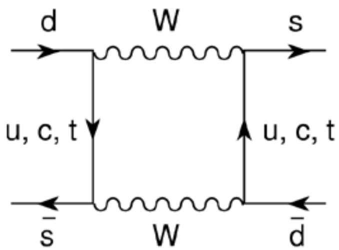
Large & Interesting Signals

Big challenge: **Get reliable SM predictions**

Severe hadronic uncertainties

INDIRECT $\not{\mathcal{CP}}$: $K^0 - \bar{K}^0$ MIXING





$$|K^0_{S,L}\rangle \sim p\,|K^0\rangle \mp q\,|\bar{K}^0\rangle$$

$$q/p \equiv (1-\bar{\varepsilon}_K)/(1+\bar{\varepsilon}_K)$$

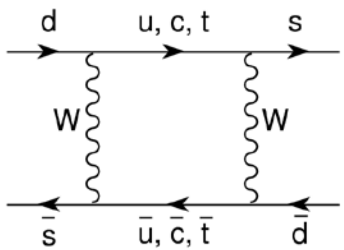
$$\boxed{\begin{gathered}\langle\bar{K}^0|\mathbf{H}|K^0\rangle \;\sim\; \sum_{ij}\lambda_i\,\lambda_j\;S(r_i,r_j)\;\eta_{ij}\quad\langle O_{\Delta S=2}\rangle \\ \langle O_{\Delta S=2}\rangle = \alpha_s(\mu)^{-2/9}\,\langle\bar{K}^0|(\bar{s}_L\gamma^\alpha d_L)(\bar{s}_L\gamma_\alpha d_L)|K^0\rangle \;\equiv\; \left(\frac{4}{3}M_K^2\,f_K^2\right)\hat{B}_K \\ \lambda_i \equiv V_{id}\,V_{is}^* \quad ; \quad r_i \equiv m_i^2/M_W^2 \qquad (i=u,c,t)\end{gathered}}$$

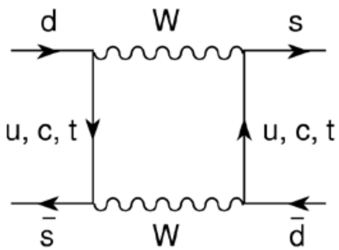
- **GIM Mechanism:** $\lambda_u + \lambda_c + \lambda_t = 0$

$$\left(M_{K_L}-M_{K_S}\right)/M_{K^0} = (7.00\pm 0.01)\cdot 10^{-15}$$

- $\mathcal{CP}$ **:** $\mathrm{Im}\,\lambda_t = -\,\mathrm{Im}\,\lambda_c \simeq \eta\,\lambda^5 A^2$
- **Hard GIM Breaking:** $S(r_i,r_i) \sim r_i$ $\Rightarrow$ **t quark**

INDIRECT $\not{\mathcal{C}}\!\!\not{\mathcal{P}}$: $K^0 - \bar{K}^0$ MIXING





$$\left|K^0_{S,L}\right\rangle \sim p\,\left|K^0\right\rangle \mp q\,\left|\bar{K}^0\right\rangle$$

$$q/p \equiv \left(1-\bar{\varepsilon}_K\right)/\left(1+\bar{\varepsilon}_K\right)$$

$$\left\langle \bar{K}^0\right|\mathbf{H}\left|K^0\right\rangle \sim \sum_{i\,j} \lambda_i\,\lambda_j\; S(r_i,r_j)\;\eta_{ij}\quad \left\langle O_{\Delta S=2}\right\rangle$$

$$\left\langle O_{\Delta S=2}\right\rangle = \alpha_s(\mu)^{-2/9}\left\langle \bar{K}^0\right|\left(\bar{s}_L\,\gamma^\alpha d_L\right)\left(\bar{s}_L\,\gamma_\alpha d_L\right)\left|K^0\right\rangle \equiv \left(\frac{4}{3}M_K^2\, f_K^2\right)\hat{B}_K$$

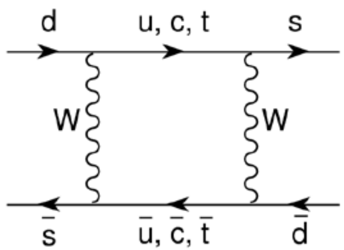
$$\lambda_i \equiv V_{id}\,V_{is}^* \qquad ; \qquad r_i \equiv m_i^2/M_W^2 \qquad (i=u,c,t)$$

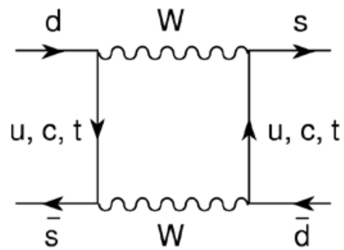
$$\mathcal{C}\,\left|K^0\right\rangle = \left|\bar{K}^0\right\rangle \quad , \quad \mathcal{P}\,\left|K^0\right\rangle = -\left|K^0\right\rangle \quad , \quad \mathcal{CP}\,\left|K^0\right\rangle = -\left|\bar{K}^0\right\rangle$$

$$\left|K^0_{1,2}\right\rangle = \frac{1}{\sqrt{2}}\left(\,\left|K^0\right\rangle \mp \left|\bar{K}^0\right\rangle\,\right) \qquad , \qquad \mathcal{CP}\,\left|K^0_{1,2}\right\rangle = \pm\,\left|K^0_{1,2}\right\rangle$$

$$\left|K^0_S\right\rangle \simeq \left|K^0_1\right\rangle + \bar{\varepsilon}_K\left|K^0_2\right\rangle \qquad , \qquad \left|K^0_L\right\rangle \simeq \left|K^0_2\right\rangle + \bar{\varepsilon}_K\left|K^0_1\right\rangle$$

INDIRECT $\not{CP}$: $K^0 - \bar{K}^0$ MIXING





$$|K^0_{S,L}\rangle \sim p\,|K^0\rangle \mp q\,|\bar{K}^0\rangle$$

$$q/p \equiv (1-\bar{\varepsilon}_K)/(1+\bar{\varepsilon}_K)$$

$$K^0 \to \pi^- l^+ \nu_l \quad (\bar{s} \to \bar{u}) \qquad ; \qquad \bar{K}^0 \to \pi^+ l^- \bar{\nu}_l \quad (s \to u)$$

$$\frac{\Gamma\left(K^0_L \to \pi^- l^+ \nu_l\right) - \Gamma\left(K^0_L \to \pi^+ l^- \bar{\nu}_l\right)}{\Gamma\left(K^0_L \to \pi^- l^+ \nu_l\right) + \Gamma\left(K^0_L \to \pi^+ l^- \bar{\nu}_l\right)} = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} = \frac{2\,\mathrm{Re}\left(\bar{\varepsilon}_K\right)}{1+|\bar{\varepsilon}_K|^2} = (0.332 \pm 0.006)\%$$

$\Rightarrow$ $\mathrm{Re}(\bar{\varepsilon}_K) = (1.66 \pm 0.03) \cdot 10^{-3}$

$$\eta_{+-} \equiv \frac{T(K_L \to \pi^+\pi^-)}{T(K_S \to \pi^+\pi^-)} \approx \varepsilon_K \qquad\qquad \eta_{00} \equiv \frac{T(K_L \to \pi^0\pi^0)}{T(K_S \to \pi^0\pi^0)} \approx \varepsilon_K$$

$$\varepsilon_K = (2.228 \pm 0.011) \cdot 10^{-3}\, e^{i\phi_\varepsilon}$$

$$\phi_\varepsilon = (43.52 \pm 0.05)^\circ$$

$\Rightarrow$ $\eta\left[(1-\rho)A^2 + 0.22\right] A^2\, \hat{B}_K = 0.143$

Buras et al

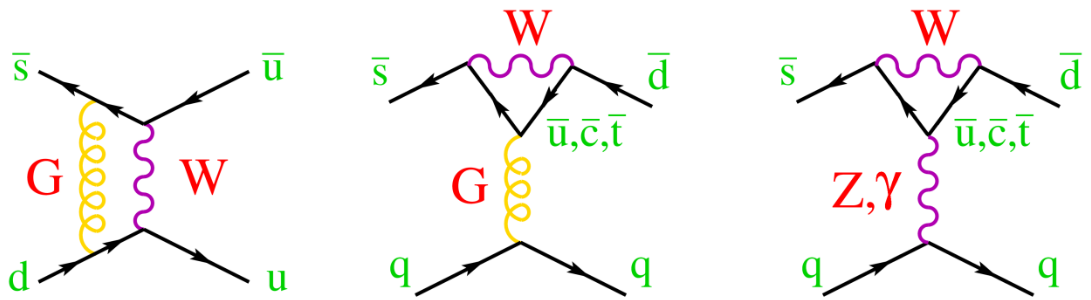
DIRECT $\not{C}\!\not{P}$ in $K \to \pi\,\pi$

$$\eta_{+-} \equiv \frac{T(K_L \to \pi^+\pi^-)}{T(K_S \to \pi^+\pi^-)} \approx \varepsilon_K + \varepsilon'_K \qquad\qquad \eta_{00} \equiv \frac{T(K_L \to \pi^0\pi^0)}{T(K_S \to \pi^0\pi^0)} \approx \varepsilon_K - 2\,\varepsilon'_K$$

$$\mathrm{Re}\left(\varepsilon'_K / \varepsilon_K\right) \approx \frac{1}{6}\left\{1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right\} = (16.6 \pm 2.3)\cdot 10^{-4}$$

NA48, NA31

KTeV, E731



$$\mathrm{Re}\left(\varepsilon'_K / \varepsilon_K\right)_{\mathrm{Th}} = \left(14 \pm 5\right)\cdot 10^{-4}$$

- Short-distance OPE

  Ciuchini et al, Buras et al

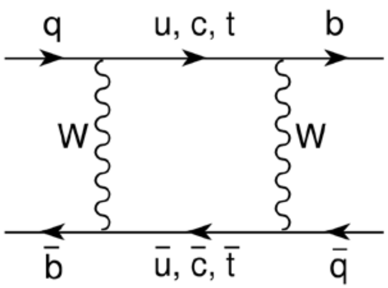
- Long-distance χPT

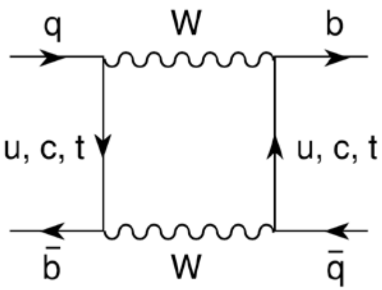
  Pallante-Pich-Scimemi

  Cirigliano-Ecker-Neufeld-Pich

  Cirigliano-Gisbert-Pich-Rodríguez

$B^0 - \bar{B}^0$ MIXING





$$V_{ud}\, V_{ub}^* \sim V_{cd}\, V_{cb}^* \sim V_{td}\, V_{tb}^* \sim A\lambda^3$$

$$\langle \bar{B}^0 | \mathrm{H} | B^0 \rangle \sim |V_{td}|^2\, S(r_t, r_t)\, \left(\frac{4}{3} M_B^2\, f_B^2\right)\, \hat{B}_B$$

$$\boxed{\Delta M_{B_d^0} = (0.5065 \pm 0.0019)\ \mathrm{ps}^{-1} \quad \Rightarrow \quad |V_{td}|}$$

- $\Delta M_{B_d^0} / \Gamma_{B_d^0} = 0.769 \pm 0.004$
- $\Delta M_{B_s^0} = (17.766 \pm 0.006)\ \mathrm{ps}^{-1}$
- $\Delta\Gamma_{B^0} / \Delta M_{B^0} \sim m_b^2 / m_t^2 \ll 1$
- $\mathrm{Re}\left(\bar{\varepsilon}_{B_d^0}\right) = -0.0005 \pm 0.0004$

$\Delta M_{B_s^0} / \Gamma_{B_s^0} = 26.89 \pm 0.11$

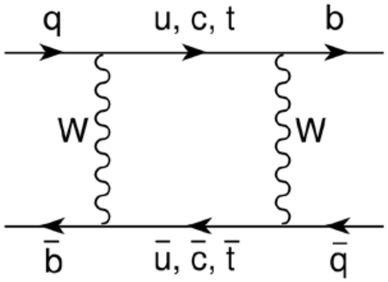
$|V_{ts}|^2 \gg |V_{td}|^2$

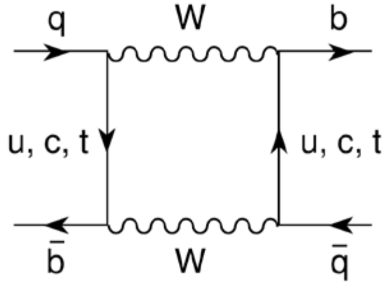
$\Delta\Gamma_{B_s^0} / \Gamma_{B_s^0} = -0.124 \pm 0.008$

$\mathrm{Re}\left(\bar{\varepsilon}_{B_s^0}\right) = -0.00015 \pm 0.00070$

$\cancel{CP}$ very small

$|q/p| - 1 \sim m_c^2 / m_t^2$





$$\mathbf{M} = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$$

$$|B^0_\mp\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \left(p\,|B^0\rangle \mp q\,|\bar{B}^0\rangle \right) \qquad , \qquad \frac{q}{p} \equiv \frac{1-\bar{\varepsilon}_B}{1+\bar{\varepsilon}_B} = \left(\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \right)^{1/2}$$

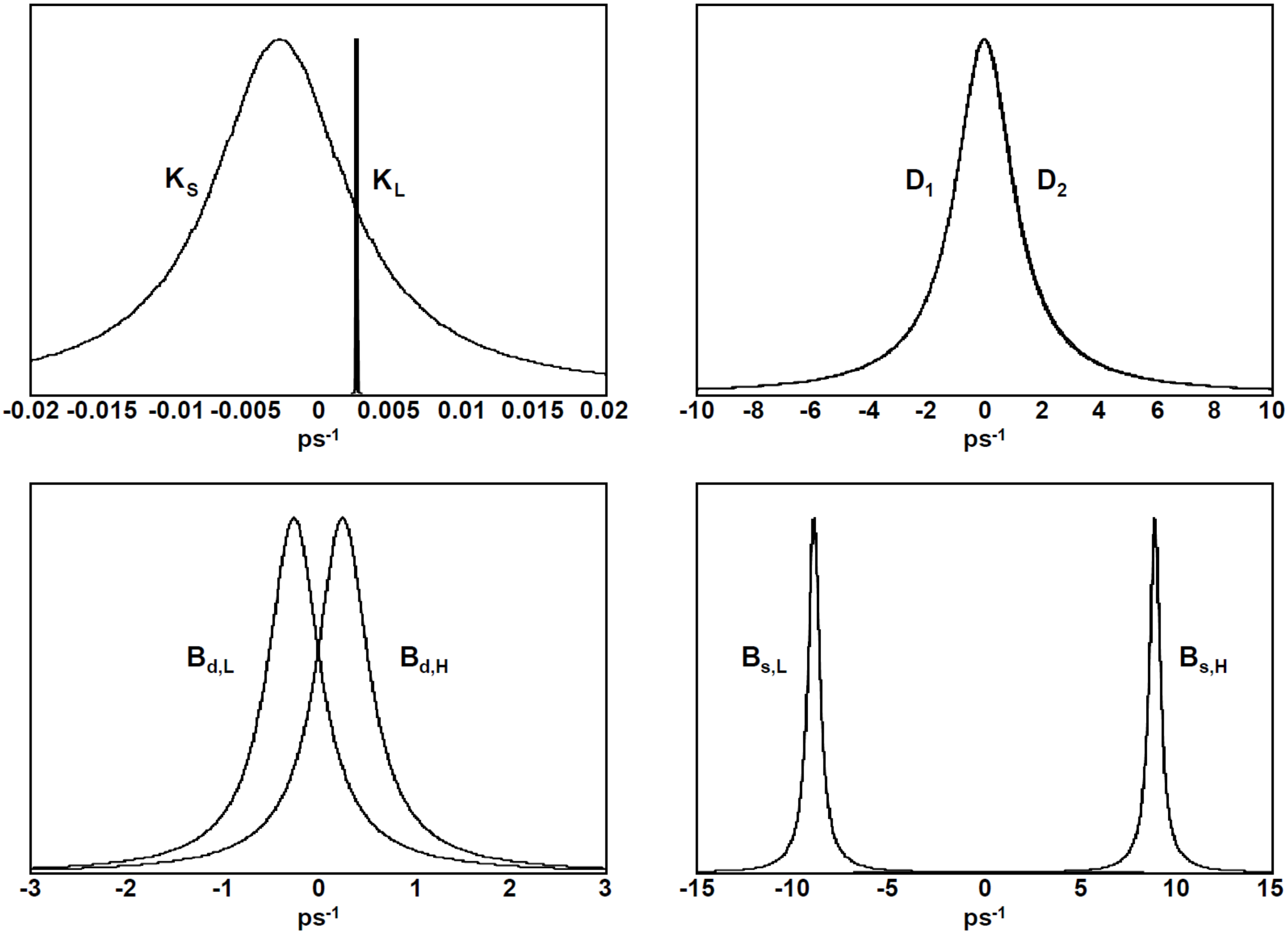
$$(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2 \qquad , \qquad \Delta M\;\Delta\Gamma = 4\,\mathrm{Re}(M_{12}\,\Gamma_{12}^*)$$

$$\Delta\Gamma/\Delta M \approx \Gamma_{12}/M_{12} \sim m_b^2/m_t^2 \ll 1 \quad \Longrightarrow \quad \left|\frac{q}{p}\right| \approx 1 + \frac{1}{2}\left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi_{\Delta B=2} \qquad , \qquad \phi_{\Delta B=2} \equiv \arg\left(M_{12}/\Gamma_{12}\right)$$

$$\Delta M \equiv M_{B_+} - M_{B_-} \qquad , \qquad \Delta\Gamma \equiv \Gamma_{B_+} - \Gamma_{B_-}$$

$$\boxed{\begin{pmatrix} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{pmatrix} = \begin{pmatrix} g_1(t) & \frac{q}{p}\,g_2(t) \\ \frac{p}{q}\,g_2(t) & g_1(t) \end{pmatrix} \begin{pmatrix} |B^0\rangle \\ |\bar{B}^0\rangle \end{pmatrix} \qquad , \qquad \begin{pmatrix} g_1(t) \\ g_2(t) \end{pmatrix} = \mathrm{e}^{-iMt}\,\mathrm{e}^{-\Gamma t/2} \begin{pmatrix} \cos\left[\left(\Delta M - \frac{i}{2}\Delta\Gamma\right)\frac{t}{2}\right] \\ -i\sin\left[\left(\Delta M - \frac{i}{2}\Delta\Gamma\right)\frac{t}{2}\right] \end{pmatrix}}$$

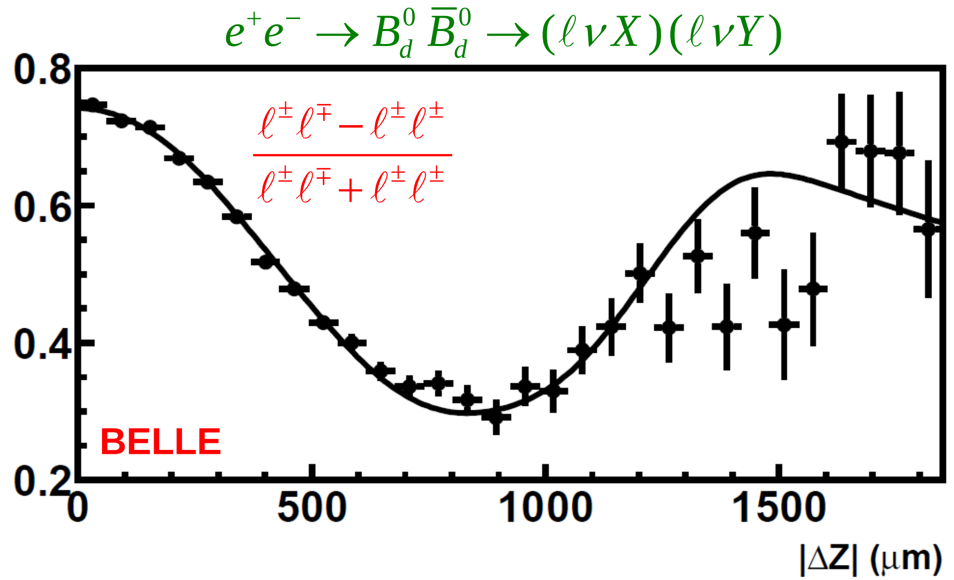
Widths & Mass Differences



M. Gersabeck

Time Scales: Oscillation ~ $\sin\left[\left(x-iy\right)\Gamma t/2\right]$

$$x \equiv \frac{\Delta M}{\Gamma} \qquad , \qquad y \equiv \frac{\Delta\Gamma}{2\Gamma}$$

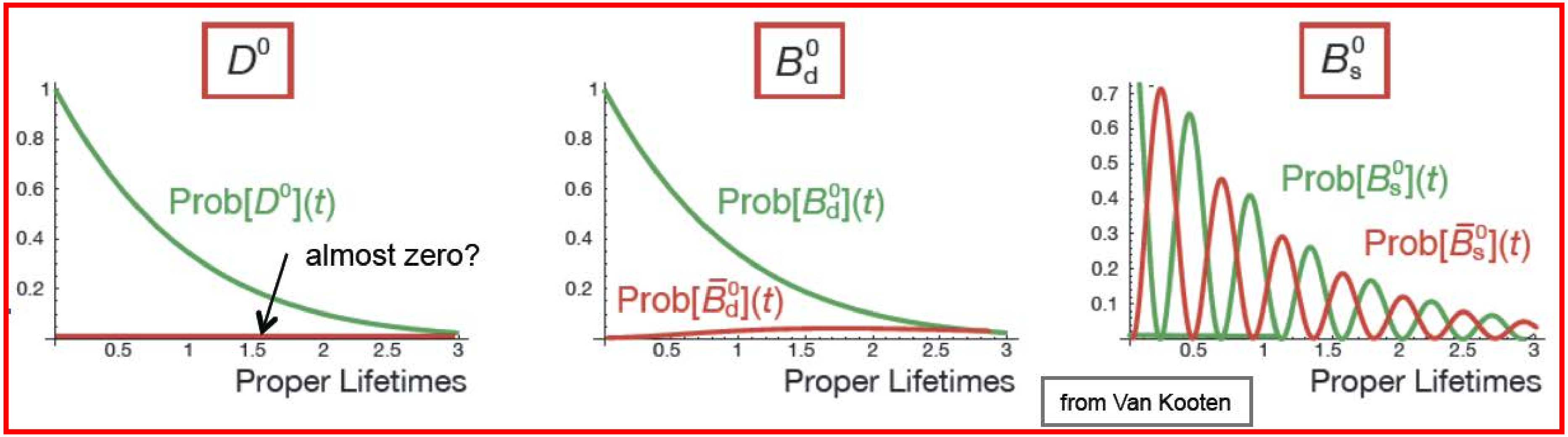


- **K⁰ :** x ~ y ~ 1
- **D⁰ :** x ~ y ~ 0.01 Slow oscillation (decays faster)
- **B_d :** x ~ 1 , y ~ 0.01
- **B_s :** x ~ 25 , y ~ 0.05 Fast oscillation (averages out to 0)

Time Scales:

Oscillation ~ $\sin\left[\left(x-iy\right)\Gamma t/2\right]$

$$x\equiv\Delta M/\Gamma \quad , \quad y\equiv\Delta\Gamma/2\Gamma$$



- **K^0 :** x ~ y ~ 1
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- **B_d :** x ~ 1 , y ~ 0.01
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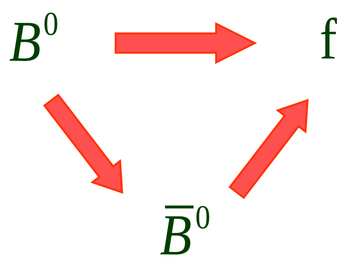
$B^0 - \bar{B}^0$ MIXING AND DIRECT $\cancel{CP}$



$$T_f \equiv T[B^0 \to f] \quad ; \quad \bar{T}_f \equiv -T[\bar{B}^0 \to f] \quad ; \quad \bar{\rho}_f \equiv \bar{T}_f / T_f$$

$$T_{\bar{f}} \equiv T[B^0 \to \bar{f}] \quad ; \quad \bar{T}_{\bar{f}} \equiv -T[\bar{B}^0 \to \bar{f}] \quad ; \quad \rho_{\bar{f}} \equiv T_{\bar{f}} / \bar{T}_{\bar{f}}$$

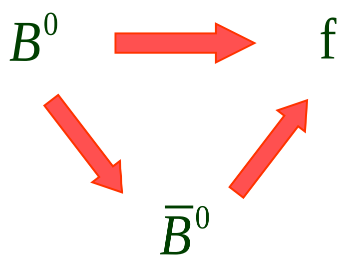
$$\mathcal{CP}\; B^0 = -\bar{B}^0 \qquad ; \qquad \mathcal{CP}\; f = \bar{f}$$

$$\boxed{\begin{aligned} \Gamma[B^0(t) \to f] &\sim \frac{1}{2}\, e^{-\Gamma t} \left(|T_f|^2 + |\bar{T}_f|^2 \right) \left\{ 1 + \mathbf{C_f}\, \cos(\Delta M\, t) - \mathbf{S_f}\, \sin(\Delta M\, t) \right\} \\ \Gamma[\bar{B}^0(t) \to \bar{f}] &\sim \frac{1}{2}\, e^{-\Gamma t} \left(|\bar{T}_{\bar{f}}|^2 + |T_{\bar{f}}|^2 \right) \left\{ 1 - \mathbf{C_{\bar{f}}}\, \cos(\Delta M\, t) + \mathbf{S_{\bar{f}}}\, \sin(\Delta M\, t) \right\} \end{aligned}}$$

$$\mathbf{C_f} \equiv \frac{1 - |\bar{\rho}_f|^2}{1 + |\bar{\rho}_f|^2} \quad ; \quad \mathbf{S_f} \equiv \frac{2\,\mathrm{Im}\left(\frac{q}{p}\, \bar{\rho}_f \right)}{1 + |\bar{\rho}_f|^2} \quad ; \quad \mathbf{C_{\bar{f}}} \equiv -\frac{1 - |\rho_{\bar{f}}|^2}{1 + |\rho_{\bar{f}}|^2} \quad ; \quad \mathbf{S_{\bar{f}}} \equiv \frac{-\,2\,\mathrm{Im}\left(\frac{p}{q}\, \rho_{\bar{f}} \right)}{1 + |\rho_{\bar{f}}|^2}$$

$$\Delta\Gamma \ll \Delta M \quad \Longrightarrow \quad \frac{q}{p} \approx \sqrt{\frac{M_{12}^*}{M_{12}}} \approx \frac{\mathbf{V}_{tb}^*\, \mathbf{V}_{tq}}{\mathbf{V}_{tb}\, \mathbf{V}_{tq}^*} = e^{-2i\phi_M} \quad ; \quad \phi_M \approx \begin{cases} \beta & (B_d^0) \\ -\beta_s \approx -\lambda^2 \eta & (B_s^0) \end{cases}$$

$B^0 - \bar{B}^0$ MIXING AND DIRECT $\not{\mathcal{CP}}$



$$T_{f} \equiv T[B^0 \to f] \quad ; \quad \bar{T}_{f} \equiv -\,T[\bar{B}^0 \to f] \quad ; \quad \bar{\rho}_{f} \equiv \bar{T}_{f} / T_{f}$$

$$T_{\bar{f}} \equiv T[B^0 \to \bar{f}] \quad ; \quad \bar{T}_{\bar{f}} \equiv -\,T[\bar{B}^0 \to \bar{f}] \quad ; \quad \rho_{\bar{f}} \equiv T_{\bar{f}} / \bar{T}_{\bar{f}}$$

$$\mathcal{CP}\; B^0 = -\,\bar{B}^0 \qquad ; \qquad \mathcal{CP}\; f = \bar{f}$$

$$\Gamma[B^0(t) \to f] \;\sim\; \frac{1}{2}\, e^{-\Gamma t} \left(|T_{f}|^2 + |\bar{T}_{f}|^2 \right) \left\{ 1 + \mathbf{C_{f}}\, \cos(\Delta M\, t) - \mathbf{S_{f}}\, \sin(\Delta M\, t) \right\}$$

$$\Gamma[\bar{B}^0(t) \to \bar{f}] \;\sim\; \frac{1}{2}\, e^{-\Gamma t} \left(|\bar{T}_{\bar{f}}|^2 + |T_{\bar{f}}|^2 \right) \left\{ 1 - \mathbf{C_{\bar{f}}}\, \cos(\Delta M\, t) + \mathbf{S_{\bar{f}}}\, \sin(\Delta M\, t) \right\}$$

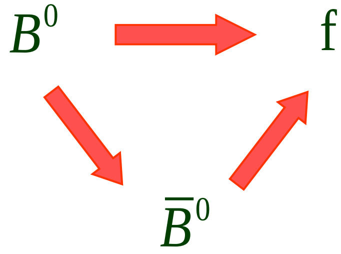
$$\mathbf{C_{f}} \equiv \frac{1 - |\bar{\rho}_{f}|^2}{1 + |\bar{\rho}_{f}|^2} \quad ; \quad \mathbf{S_{f}} \equiv \frac{2\,\mathrm{Im}\left(\frac{q}{p}\, \bar{\rho}_{f} \right)}{1 + |\bar{\rho}_{f}|^2} \quad ; \quad \mathbf{C_{\bar{f}}} \equiv -\,\frac{1 - |\rho_{\bar{f}}|^2}{1 + |\rho_{\bar{f}}|^2} \quad ; \quad \mathbf{S_{\bar{f}}} \equiv \frac{-\,2\,\mathrm{Im}\left(\frac{p}{q}\, \rho_{\bar{f}} \right)}{1 + |\rho_{\bar{f}}|^2}$$

CP self-conjugate: $\bar{f} = \eta_{f}\; f$ $\Longrightarrow$

$$T_{\bar{f}} = \eta_{f}\; T_{f} \quad ; \quad \bar{T}_{\bar{f}} = \eta_{f}\; \bar{T}_{f} \quad ; \quad \rho_{\bar{f}} \equiv 1/\bar{\rho}_{f}$$

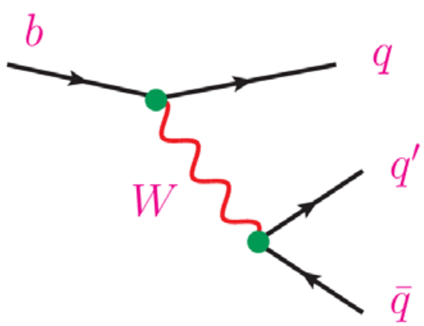
$$\mathbf{C_{\bar{f}}} = \mathbf{C_{f}} \qquad ; \qquad \mathbf{S_{\bar{f}}} = \mathbf{S_{f}}$$

$B^0 - \bar{B}^0$ MIXING AND DIRECT $\cancel{CP}$



CP self-conjugate: $\bar{\mathrm{f}} = \eta_{\mathrm{f}}\, \mathrm{f}$

$$\frac{q}{p} \approx \frac{\mathbf{V}_{tb}^* \, \mathbf{V}_{tq}}{\mathbf{V}_{tb} \, \mathbf{V}_{tq}^*} = e^{-2i\phi_M} \qquad ; \qquad \phi_M \approx \begin{cases} \beta & \left(B_d^0\right) \\ -\beta_s \approx -\lambda^2 \eta & \left(B_s^0\right) \end{cases}$$



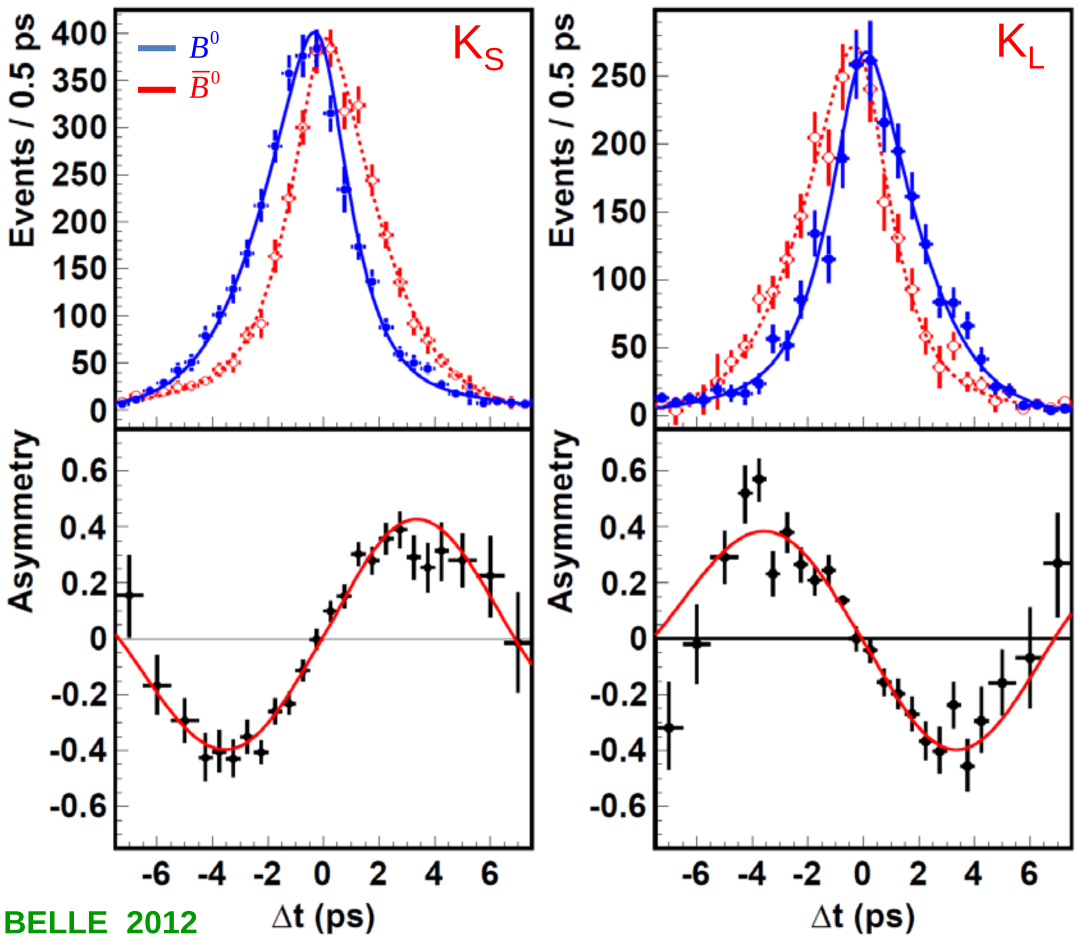
Assumption: **Only 1 decay amplitude**

$$\frac{A_{b\to q\bar{q}q'}}{A_{\bar{b}\to \bar{q}q\bar{q}'}} = \frac{\mathbf{V}_{qb} \, \mathbf{V}_{qq'}^*}{\mathbf{V}_{qb}^* \, \mathbf{V}_{qq'}} = e^{-2i\phi_D} \quad \Longrightarrow \quad \rho_{\bar{f}} = \bar{\rho}_f^* = \eta_f \, e^{2i\phi_D} \qquad C_f = 0$$

$$\Longrightarrow \quad \frac{\Gamma\left(\bar{B}^0 \to \bar{\mathrm{f}}\right) - \Gamma\left(B^0 \to \mathrm{f}\right)}{\Gamma\left(\bar{B}^0 \to \bar{\mathrm{f}}\right) + \Gamma\left(B^0 \to \mathrm{f}\right)} = -\eta_{\mathrm{f}} \, \sin(2\phi)\, \sin(\Delta M\, t) \qquad ; \qquad \phi = \phi_M + \phi_D$$

Direct information on the CKM matrix

$$\frac{\Gamma(\bar{B}^0 \to J/\psi\, K_S) - \Gamma(B^0 \to J/\psi\, K_S)}{\Gamma(\bar{B}^0 \to J/\psi\, K_S) + \Gamma(B^0 \to J/\psi\, K_S)} \;=\; -\eta_f\; \sin(2\beta)\; \sin(\Delta M\, t)$$

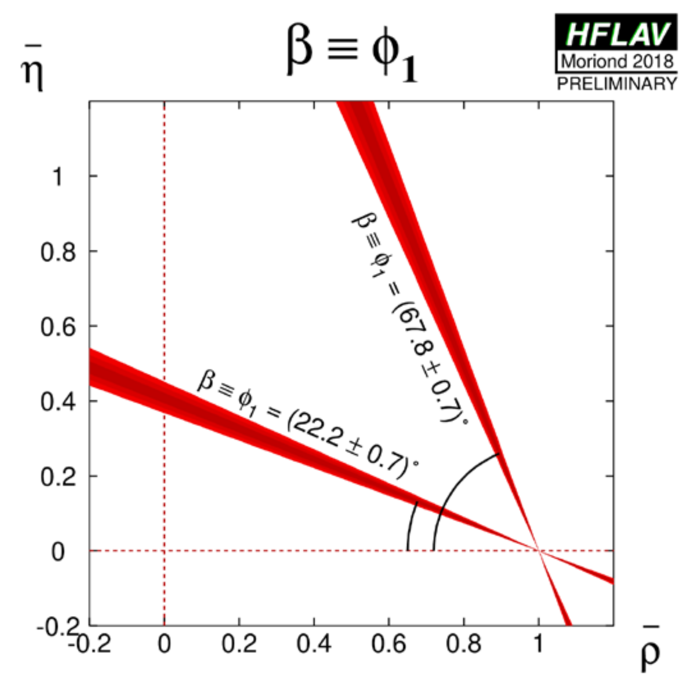


BELLE 2012

PDG 2020:

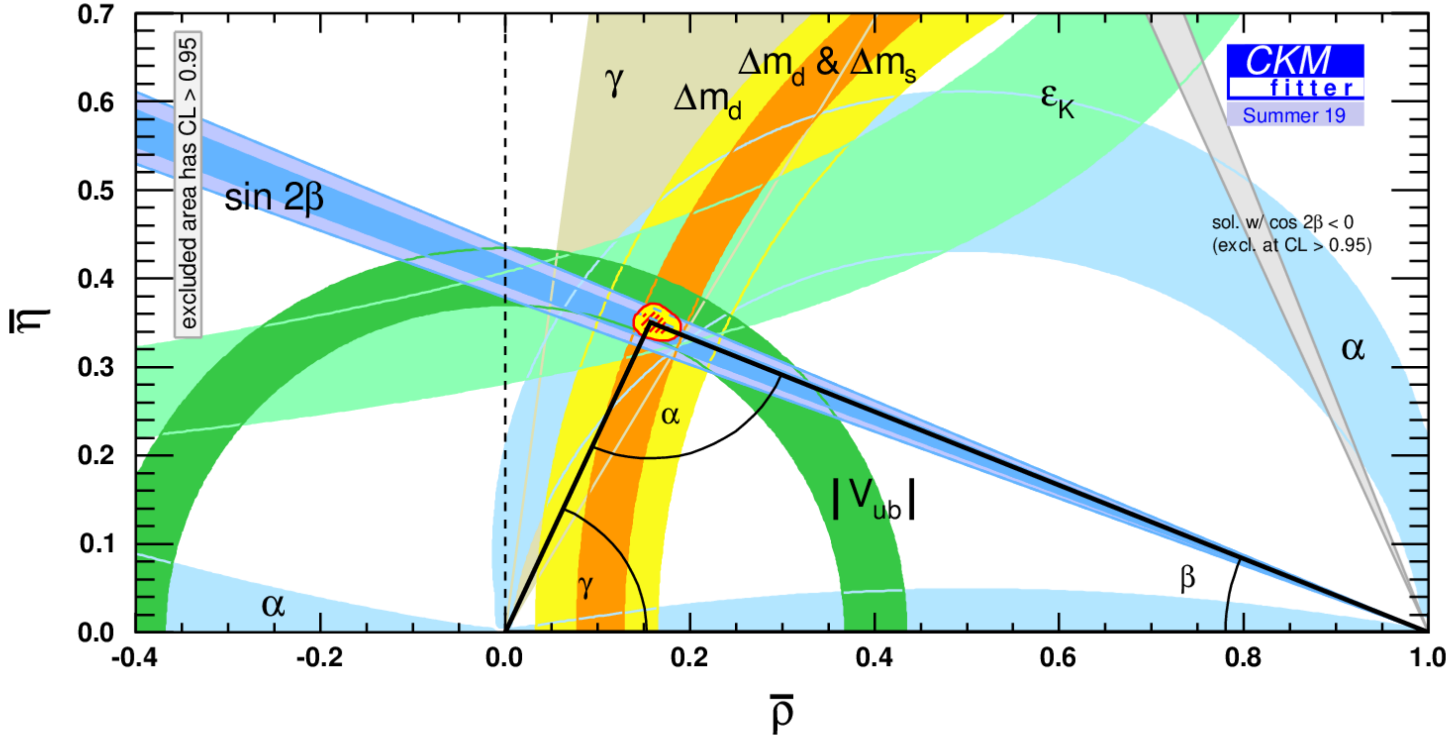
sin(2β) = 0. 701 ± 0.017

$B^0 \to J/\psi\, K_{S,L}\,,\, \psi(2S)\, K_S\,,\, \chi_c\, K_S\,,\, \eta_c\, K_S$

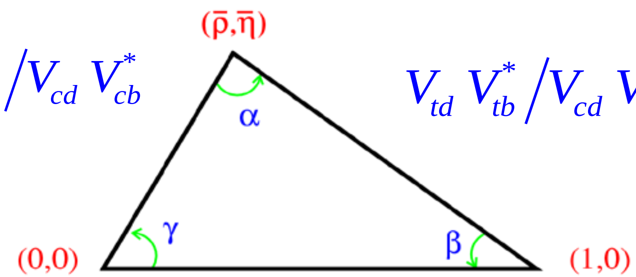


$\not{CP}$ Signal

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



$V_{ud}V_{ub}^* / V_{cd}V_{cb}^*$



$V_{td}V_{tb}^* / V_{cd}V_{cb}^*$

UT*fit*

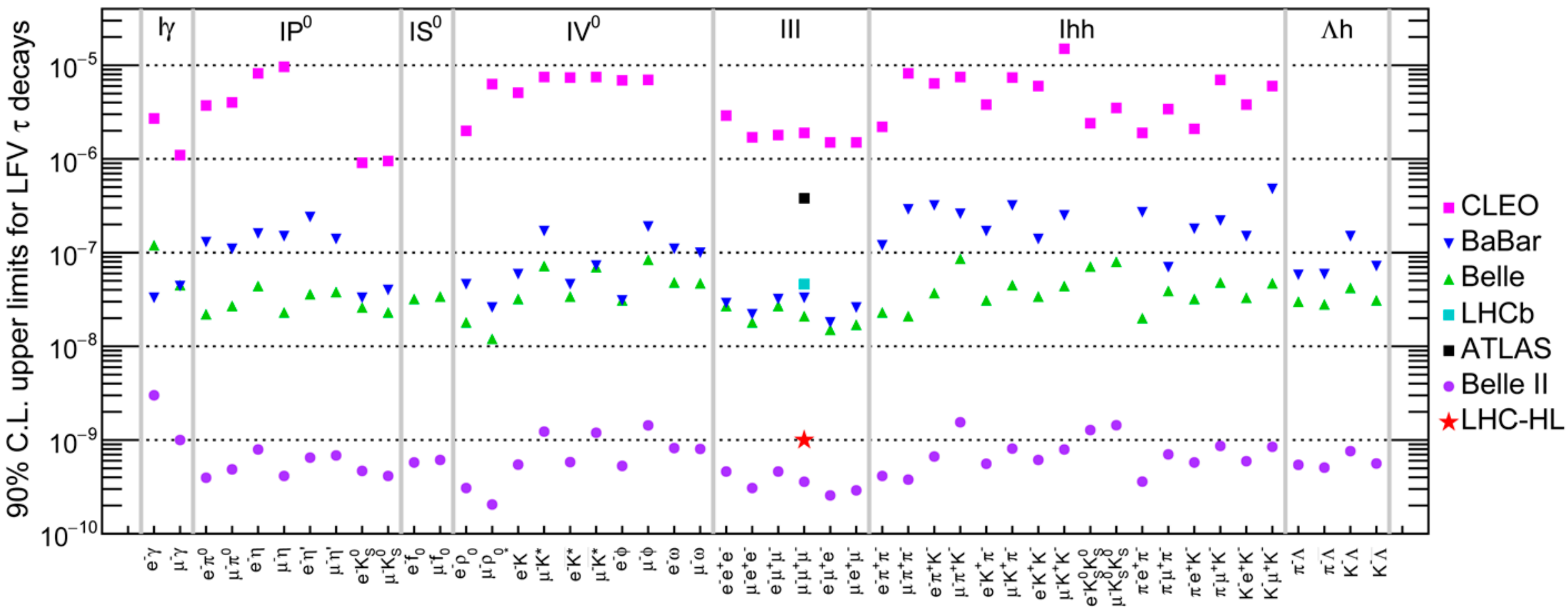
$$\bar{\eta} \equiv \eta\left(1-\frac{1}{2}\lambda^2\right) = 0.348 \pm 0.010$$

$$\bar{\rho} \equiv \rho\left(1-\frac{1}{2}\lambda^2\right) = 0.148 \pm 0.013$$

$\alpha = 90.1 \pm 2.2°$; $\beta = 23.8 \pm 1.3°$; $\gamma = 65.8 \pm 2.2°$

Bounds on Lepton Flavour Violation

τ Decays (90% CL)



$\mathrm{Br}(\mu \to e\gamma) < 4.2 \times 10^{-13}$ (MEG, 90% CL) , $\mathrm{Br}(\mu \to 3e) < 1.0 \times 10^{-12}$ (SINDRUM, 90% CL)

$\mathrm{Br}(K_L \to \mu e) < 4.7 \times 10^{-12}$ (BNL-E871, 90% CL) , $\mathrm{Br}(K^+ \to \pi^+\mu^+e^-) < 1.3 \times 10^{-11}$ (BNL-E865, 90% CL)

$\mathrm{Br}(B^0 \to e\mu) < 1.0 \times 10^{-9}$ (LHCb, 90% CL) , $\mathrm{Br}(D^0 \to e\mu) < 1.3 \times 10^{-8}$ (LHCb, 90% CL)

$\mathrm{Br}(Z^0 \to e\mu) < 7.5 \times 10^{-7}$ (ATLAS, 95% CL) , $\mathrm{Br}(H \to e\mu) < 6.1 \times 10^{-5}$ (ATLAS, 95% CL)

$\mathrm{Br}(Z^0 \to e\tau) < 5.0 \times 10^{-6}$ (ATLAS, 95% CL) , $\mathrm{Br}(H \to e\tau) < 4.7 \times 10^{-3}$ (CMS, 95% CL)

$\mathrm{Br}(Z^0 \to \mu\tau) < 6.5 \times 10^{-6}$ (ATLAS, 95% CL) , $\mathrm{Br}(H \to \mu\tau) < 2.5 \times 10^{-3}$ (ATLAS, 95% CL)

SUMMARY

- **Flavour Structure and** $\not{CP}$ are major pending questions
- **Related to SSB** → **Scalar Sector** (Higgs)
- Important **cosmological implications** (Baryogenesis)
- Sensitive to **New Physics:** **Flavour Anomalies!**

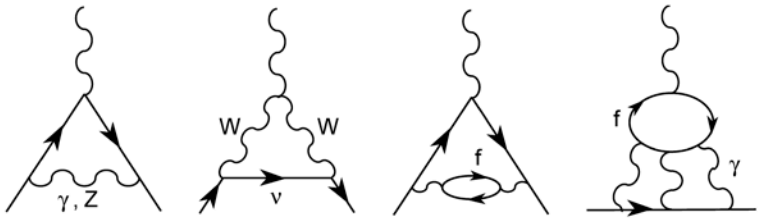
Intriguing signals **(most anomalies related to 3rd family)**

Many questions. Higher statistics & better systematics (QCD) needed

Eagerly awaiting new experimental results

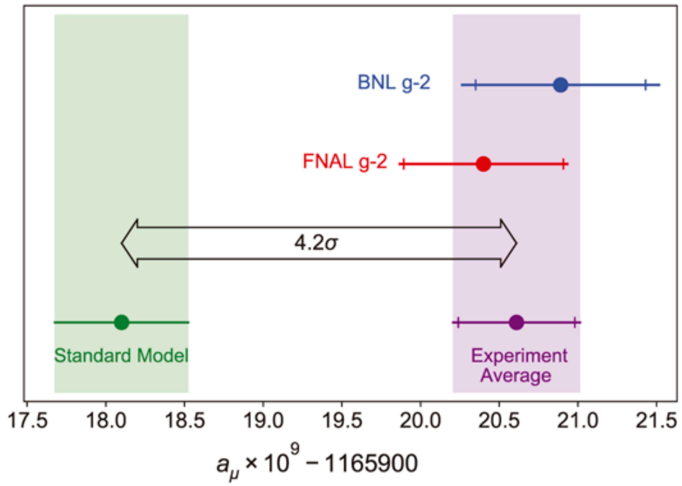
Backup

μ Anomalous Magnetic Moment



White Paper (2020) — G. Colangelo, Moriond EW 2021

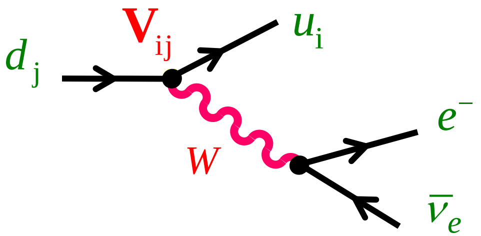
Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	$-98.3(7)$
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice , *udsc*)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, *uds*)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment (E821)	116 592 089(63)
Difference: $\Delta a_\mu := a_\mu^{\rm exp} - a_\mu^{\rm SM}$	279(76)



$$\Delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = (251 \pm 59)\cdot 10^{-11} \qquad (4.2\sigma)$$

Much smaller discrepancy with the SM prediction obtained from BMW (lattice) or τ data

V_{ij} Determination $(0^- \to 0^-)$



$K \to \pi \ell \nu$, $D \to K \ell \nu \ldots$

$$\langle P'(k') | \bar{u}_i \gamma^\mu d_j | P(k) \rangle = C_{PP'} \left\{ (k+k')^\mu \; f_+(q^2) + (k-k')^\mu \; f_-(q^2) \right\}$$

$$\Gamma(P \to P' l \nu) = \frac{G_F^2 M_P^5}{192\pi^3} \, |\mathbf{V_{ij}}|^2 \, C_{PP'}^2 \, |f_+(0)|^2 \, \mathbf{I} \, (1+\delta_{\mathrm{RC}})$$

$$\mathbf{I} \approx \int_0^{(M_P - M_{P'})^2} \frac{dq^2}{M_P^8} \, \lambda^{3/2}(q^2, M_P^2, M_{P'}^2) \left| \frac{f_+(q^2)}{f_+(0)} \right|^2$$

$f_-(q^2)$ suppressed

$(k-k')^\mu \; \bar{l} \gamma_\mu (1-\gamma_5) \nu_l \sim m_l$

- Measure the q^2 distribution $\Rightarrow$ $\mathbf{I}$
- Measure Γ $\Rightarrow$ $f_+(0) \, |\mathbf{V_{ij}}|$
- Get a theoretical prediction for $f_+(0)$ $\Rightarrow$ $|\mathbf{V_{ij}}|$

Theory is always needed: Symmetries

PDG parametrization of the CKM matrix

$$\mathbf{V} = \begin{bmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta_{13}} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & c_{23}\, c_{13} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Wolfenstein: $s_{12} \equiv \lambda \quad , \quad s_{23} \equiv A\lambda^2 \quad , \quad s_{13}\, e^{-i\delta_{13}} \equiv A\lambda^3(\rho - i\eta)$

$$\mathbf{V} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3\,(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\,\lambda^2 \\ A\lambda^3\,(1-\rho-i\,\eta) & -A\,\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

Standard Model $\not{\mathcal{CP}}$: 3 fermion families needed

$$\not{\mathcal{CP}} \quad \Longleftrightarrow \quad \mathbf{H}(M_u^2) \cdot \mathbf{H}(M_d^2) \cdot \mathbf{J} \neq 0$$

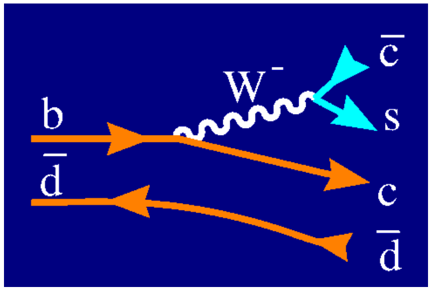
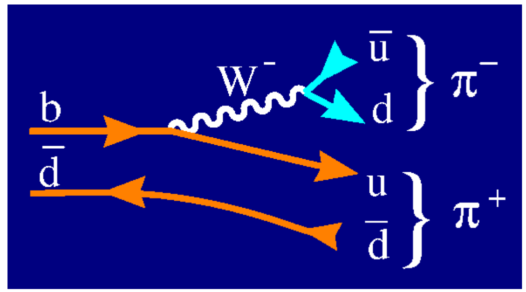
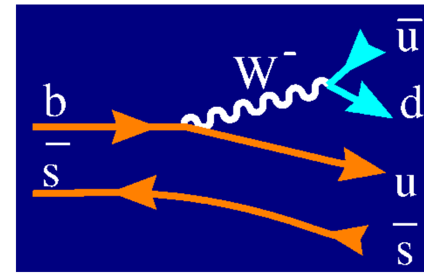
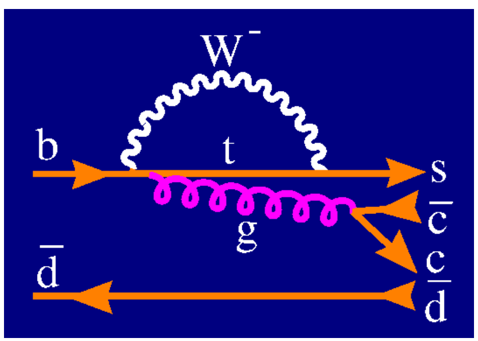
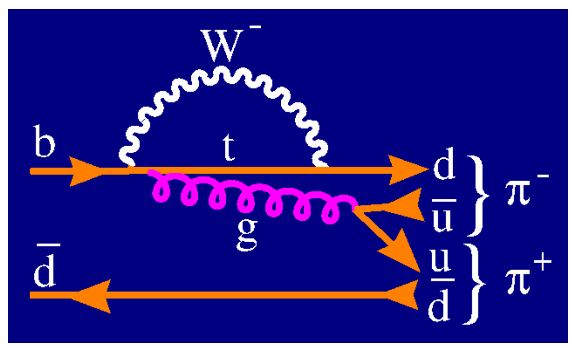
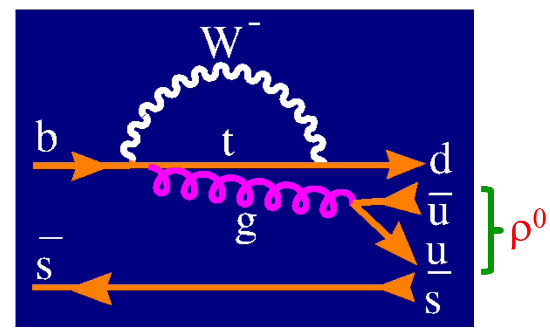
$$\mathbf{H}(M_u^2) \equiv (m_t^2 - m_c^2)\,(m_c^2 - m_u^2)\,(m_t^2 - m_u^2)$$

$$\mathbf{H}(M_d^2) \equiv (m_b^2 - m_s^2)\,(m_s^2 - m_d^2)\,(m_b^2 - m_d^2)$$

$$\mathbf{J} = c_{12}\, c_{13}^2\, c_{23}\, s_{12}\, s_{13}\, s_{23} \sin\delta_{13} = \left| A^2 \lambda^6 \eta \right| < 10^{-4}$$

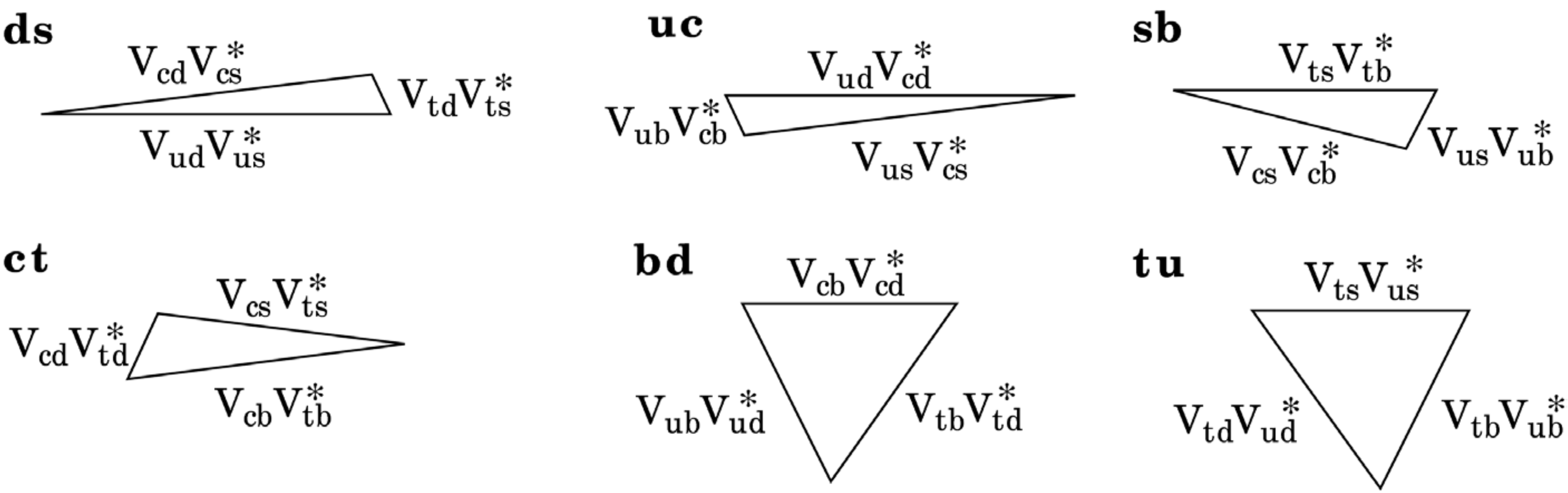
$\Rightarrow$

- Low-Energy Phenomena
- Small Effects ~ **J**
- Big Asymmetries $\Longleftrightarrow$ Suppressed Decays
- B Decays are an optimal place for $\not{\mathcal{CP}}$ signals

$\bar{B}_d^0 \to J/\Psi\, K_S^0$	$\bar{B}_d^0 \to \pi^+\pi^-$	$\bar{B}_s^0 \to \rho^0 K_S^0$
$\phi \simeq \beta$	$\phi \simeq \beta + \gamma = \pi - \alpha$	$\phi \neq \gamma$
 $V_{cb}\, V_{cs}^* \;\sim A\lambda^2$	 $V_{ub}\, V_{ud}^* \;\sim\; A\lambda^3(\rho - i\eta)$	 $V_{ub}\, V_{ud}^* \;\sim\; A\lambda^3(\rho - i\eta)$
 $V_{tb}\, V_{ts}^* \;\sim\; -A\lambda^2$	 $V_{tb}\, V_{td}^* \;\sim\; A\lambda^3(1-\rho+i\eta)$	 $V_{tb}\, V_{td}^* \;\sim\; A\lambda^3(1-\rho+i\eta)$
***	**	**BAD**

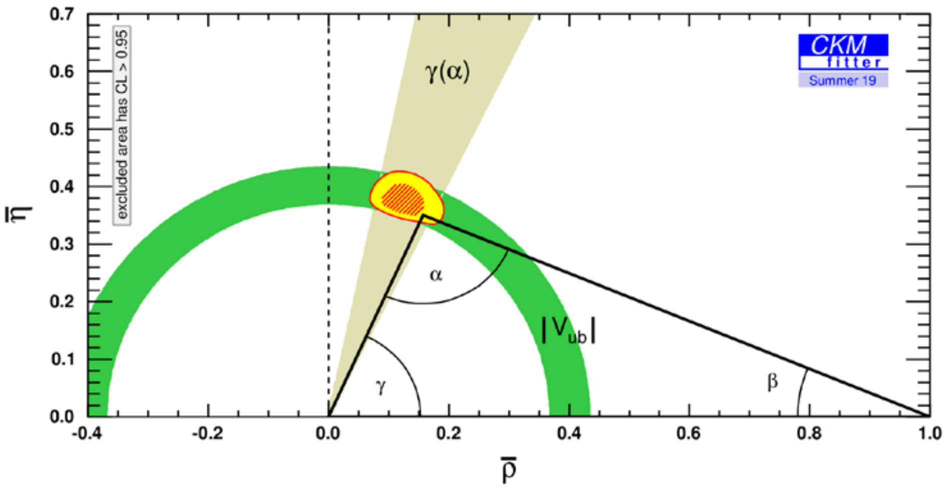
UNITARITY TRIANGLES

$$\mathbf{V_{ui}\,V_{uj}^* + V_{ci}\,V_{cj}^* + V_{ti}\,V_{tj}^* = 0} \qquad (i \neq j)$$

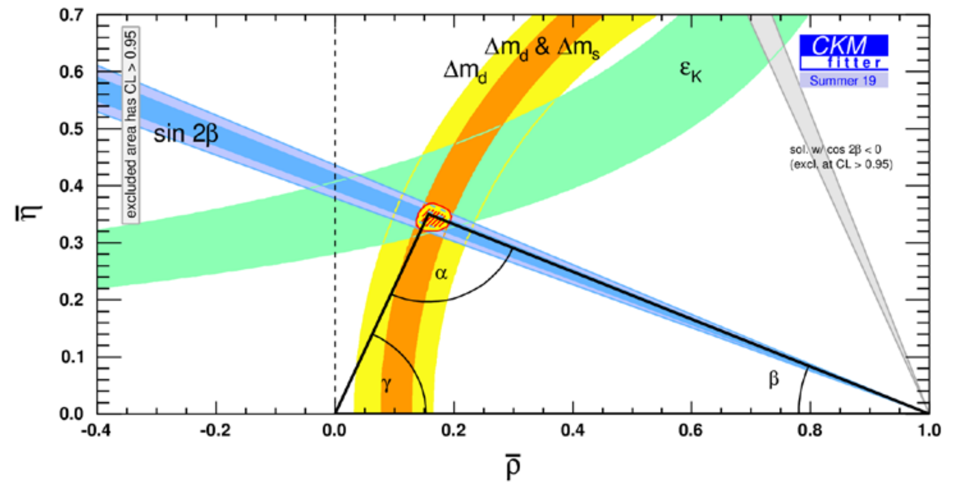


$$\mathbf{V} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

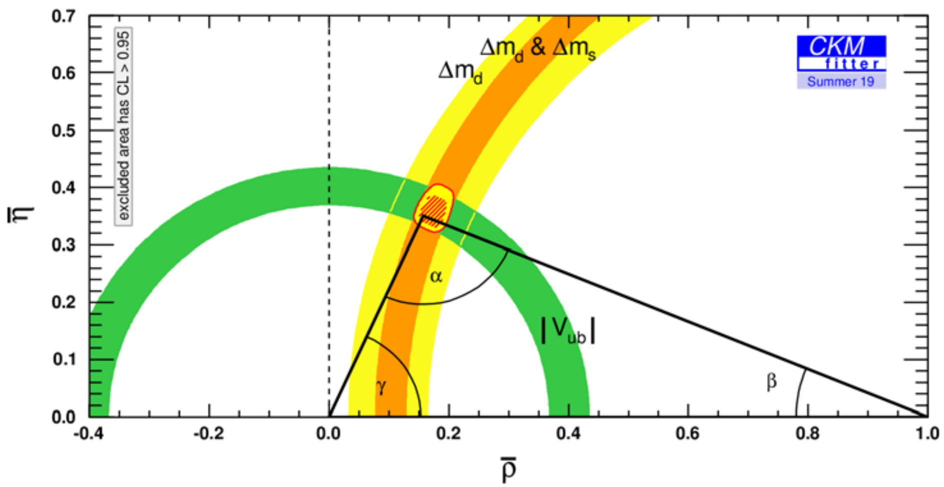
Tree-level determinations



Loop processes



CP Conserving



CP Violating